

Problems: Work, Energy, Power

- 1) A 10.0 kg mass sliding on a frictionless horizontal surface at 7.00 m/s hits a spring that is attached to a wall. The spring has a spring constant of 5000 N/m.

- a) Determine the maximum compression of the spring.

At maximum compression, the box has a speed of zero. Therefore, comparing just before it hits the spring to the point of maximum compression...

$$\cancel{U_i} + K_i + \cancel{W_{nc}} = U_f + \cancel{K_f}$$

$$\frac{1}{2}mv_i^2 = \frac{1}{2}kx_f^2$$

$$x_f = \sqrt{\frac{mv_i^2}{k}} \quad x_f = \sqrt{\frac{(10 \text{ kg})(7\frac{\text{m}}{\text{s}})^2}{5000 \text{ N/m}}} \quad x_f = 0.313 \text{ m}$$

- b) Determine the speed of the box in the above problem when the spring had a compression of 0.100 m.

As the spring is compressing, the box is still in motion (and so still has some kinetic energy).

$$\frac{1}{2}mv_i^2 = \frac{1}{2}mv_f^2 + \frac{1}{2}kx_f^2$$

$$mv_f^2 = mv_i^2 - kx_f^2$$

$$v_f = \sqrt{\frac{mv_i^2 - kx_f^2}{m}} = \sqrt{\frac{(10 \text{ kg})(7\frac{\text{m}}{\text{s}})^2 - (5000\frac{\text{N}}{\text{m}})(0.1 \text{ m})^2}{10 \text{ kg}}} = 6.63 \text{ m/s}$$

- c) Suppose the box springs back off of the spring with no loss of energy, after sliding on the frictionless surface, it encounters another level surface with a coefficient of friction of 0.200. How far does the box slide on this surface before coming to rest?

No mechanical energy is lost during spring compression/decompression, so you can compare any point in time to the final point when the box has been brought to rest by friction. Comparing the point before the box hits the spring (or right after it bounces off the spring), you have to realize there will now be negative non-conservative work done by friction...

$$\frac{1}{2}mv_i^2 + W_{nc} = 0$$

There is no mechanical energy left at the end, it has all been converted to thermal.

$$\frac{1}{2}mv_i^2 + fd\cos 180^\circ = 0$$

Cos(180°) = -1, so...

$$\frac{1}{2}mv_i^2 = fd$$

$$f = \mu mg$$

$$\frac{1}{2}mv_i^2 = \mu mgd$$

Solving for d...

$$d = \frac{v_i^2}{2\mu g} = \frac{(7\text{m/s})^2}{2(0.2)(9.8\frac{\text{m}}{\text{s}^2})} = 12.5 \text{ m}$$

- 2) A human cannonball uses a spring-loaded cannon to launch himself. He has a mass of 70 kg, and the spring compresses a total distance of 1.40 m. If he needs a launch velocity of 18.0 m/s, what should be the value of the spring's spring constant?

Before launch, all the energy is stored in spring potential energy. When he comes out of the cannon, all energy is kinetic (assuming he was launched horizontally).

$$\frac{1}{2}kx_i^2 = \frac{1}{2}mv_f^2$$

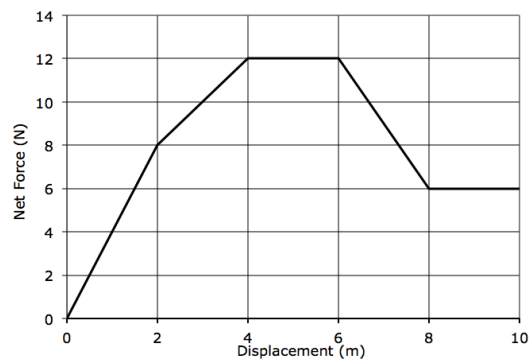
$$k = \frac{mv_f^2}{x_i^2} = \frac{(70 \text{ kg})(18 \frac{\text{m}}{\text{s}})^2}{(1.4 \text{ m})^2} = 11,600 \text{ N/m}$$

- 3) A 12.0 kg mass is hung from a spring with a spring constant of 2400 N/m. How much did the spring stretch from its equilibrium position?

This is not an energy problem, it is simply a spring in equilibrium. There are just two forces on the mass, gravity (down) and spring (up).

$$mg = kx$$

$$x = \frac{mg}{k} = \frac{(12 \text{ kg})(9.8 \frac{\text{m}}{\text{s}^2})}{2400 \frac{\text{N}}{\text{m}}} = 0.049 \text{ m}$$



- 4) A 6.00 kg mass is acted on by a net force shown in the graph above. a) if the object started from rest, what is its velocity after it has moved 10.0 m? b) If the object in initially was moving at 5.00 m/s in the direction of the net force, what was its speed after it had moved a distance of 6.00 m?

The work done on the object during each 2.00 s interval can be calculated using $W = F_{\text{average}} d$, which is equivalent to calculating the area under the graph for that section. The work done in each interval; is as follows:

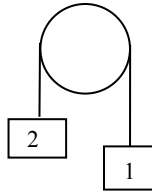
0 – 2 s:	$F_{\text{average}} = (0\text{N} + 8\text{N})/2 = 4 \text{ N}$	$d = 2 \text{ m}$	Work = 8 J
2 – 4 s:	$F_{\text{average}} = (8\text{N} + 12\text{N})/2 = 10 \text{ N}$	$d = 2 \text{ m}$	Work = 20 J
4 – 6 s:	$F_{\text{average}} = (12\text{N} + 12\text{N})/2 = 12 \text{ N}$	$d = 2 \text{ m}$	Work = 24 J
6 – 8s:	$F_{\text{average}} = (12\text{N} + 6\text{N})/2 = 9 \text{ N}$	$d = 2 \text{ m}$	Work = 18 J
8 – 10 s:	$F_{\text{average}} = (6\text{N} + 6\text{N})/2 = 6 \text{ N}$	$d = 2 \text{ m}$	Work = 12 J

a) During the 10.0 m of displacement a total of 82.0 J of work was done on the object.

$$W_{\text{net}} = \Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 \quad v = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2(82 \text{ J})}{6 \text{ kg}}} = 5.23 \text{ m/s}$$

b) During the first 6.0 m of displacement a total of 52 J of work was done of the object.

$$W_{\text{net}} = \Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 \quad v_f = \sqrt{\frac{2W + mv_i^2}{m}} = \sqrt{\frac{2(52 \text{ J}) + (6 \text{ kg})(5 \frac{\text{m}}{\text{s}})^2}{6 \text{ kg}}} = 6.51 \text{ m/s}$$



- 5) In the diagram above the pulley is frictionless. Mass 1 is 3.00 kg and mass 2 is 7.00 kg, and the two masses are released from rest. After the two masses have each moved 0.500 m, what are their velocities?

In this problem you need to keep track of the kinetic and potential energies of each object. Both objects start from rest ($K = 0$) and then gain kinetic energy. The lighter box goes up, so it gains U_g . The heavier box goes down, so it loses U_g . You can choose different points for $U_g = 0$ for each box. I chose to call the lowest point each box reaches its $U_g = 0$. This is the starting point for the lighter box (because it goes up) and the ending point for the heavy box (because it goes down).

$$m_2 g y_{2i} = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2 + m_1 g y_{1f}$$

Factor out v_f and isolate it...

$$v_f = \frac{2(m_2 g y_{2i} - m_1 g y_{1f})}{(m_1 + m_2)} = 1.98 \frac{m}{s}$$

- 6) A mass is dropped from a height of 4.00 m onto a spring with a spring constant of 120 N/m. If the spring compresses by a maximum amount of 1.25 m, what is the mass of the object?

In this problem, the object starts with just gravitational potential energy and ends with all the energy in elastic potential energy in the spring.

$$m g y_i = \frac{1}{2} k x^2$$

Solving for m ,

$$m = \frac{k x^2}{2 g y_i}$$

The only tricky thing is using the correct y_i . Remember, the box falls 4 meters before hitting the spring and then moves down an additional 1.25 m while the spring is compressing. The total distance down is therefore 4 m + 1.25 m = 5.25 m.

$$m = \frac{k x^2}{2 g y_i} = \frac{\left(120 \frac{N}{m}\right) (1.25 \text{ m})^2}{2 \left(9.8 \frac{m}{s^2}\right) (5.25 \text{ m})} = 1.82 \text{ kg}$$

- 7) What was the speed of the mass in the above problem when the spring was compressed a distance of 0.500 m?

When the spring was partially compressed, the mass was still moving.

$$m g y_i = \frac{1}{2} m v^2 + \frac{1}{2} k x^2$$

Solve for v_f ...

$$v_f = \sqrt{\frac{2 m g y_i - k x^2}{m}} = 8.46 \frac{m}{s}$$

- 8) A 10.0 kg mass is dropped from a tall building. During the first second of the fall, what was the average power exerted by gravity? What was the average power exerted by gravity during the first 5.00 seconds of the fall?

You need to first figure out how far the ball falls in 1.0 second and in 5.0 seconds...

$$\Delta y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$$

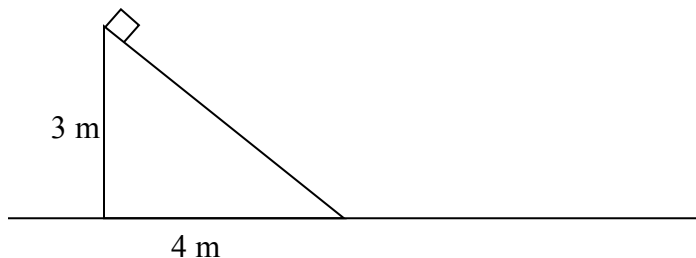
but, $y_0 = 0$ and $v_{0y} = 0$.

$$\Delta y = \frac{1}{2} \left(9.8 \frac{m}{s^2} \right) (1.0 \text{ s})^2 = 4.90 \text{ m (during first 1.0 second)}$$

$$\Delta y = \frac{1}{2} \left(9.8 \frac{m}{s^2} \right) (5.0 \text{ s})^2 = 123.5 \text{ m (during first 5.0 seconds)}$$

$$\bar{P} = \frac{\Delta E}{t} = \frac{mgy}{t} = \frac{(10 \text{ kg})(9.8 \frac{m}{s^2})(4.90 \text{ m})}{1 \text{ s}} = 480 \text{ W (during first 1.0 s)}$$

$$\bar{P} = \frac{\Delta E}{t} = \frac{mgy}{t} = \frac{(10 \text{ kg})(9.8 \frac{m}{s^2})(123.5 \text{ m})}{5 \text{ s}} = 2400 \text{ W (during first 5.0 s)}$$



- 9) A 2.0 kg box slides down a frictionless incline as shown above. Determine the following:

A) As the box slides, determine the amount of work done by:

- i) the incline (normal force)

$$W = Fd \cos \Theta \quad (\Theta = 90^\circ)$$

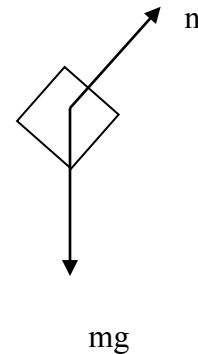
$$W = 0$$

- ii) gravity

$$W = Fd \cos \Theta \quad (F = mg; \quad \cos \Theta = 3/5; \quad d = 5 \text{ m})$$

$$W = (mg)(y)$$

$$W = (2 \text{ kg})(9.8 \text{ m/s}^2)(3 \text{ m}) = 58.8 \text{ J}$$



B) What is the net work done on the box as it slides?

$$W = W_1 + W_2 = 58.8 \text{ J}$$

C) What is the final kinetic energy of the box (at the bottom of the incline)?

$$W_{\text{net}} = \Delta K$$

$$W_{\text{net}} = K_f - K_i$$

$$K_f = 58.8 \text{ J}$$

D) What is the final velocity of the box at the bottom of the incline?

$$K = \frac{1}{2} mv^2$$

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(58.8 \text{ J})}{2 \text{ kg}}} = 7.67 \text{ m/s}$$

10) Now let's say that there is friction. As the box slides, friction exerts a force of 3.00 N opposite its motion.

A) As the box slides, determine the amount of work done by:

i) the incline (normal force)

$$W = 0 \text{ (same as in \#1)}$$

ii) gravity

$$W = 58.8 \text{ J (same as in \#1)}$$

iii) friction

$$W = Fd \cos \Theta$$

$$F = 3.00 \text{ N}; \quad \Theta = 180^\circ; \quad d = 5 \text{ m}$$

$$W = -15 \text{ J}$$

B) What is the net work done on the box as it slides?

$$W = W_1 + W_2 + W_3 = 0 + 58.8 \text{ J} + (-15 \text{ J})$$

$$W = 43.8 \text{ J}$$

C) What is the final kinetic energy of the box (at the bottom of the incline)?

$$W_{\text{net}} = \Delta KE = KE_f - KE_i$$

$$KE_f = 43.8 \text{ J}$$

D) What is the final velocity of the box at the bottom of the incline?

$$KE = (1/2)mv^2$$

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(43.8 \text{ J})}{2 \text{ kg}}} = 6.62 \text{ m/s}$$

11) A projectile is shot off a cliff 70.0 m high at an angle of 35° above the horizontal. It has an initial velocity of 26.0 m/s. What is the projectile's speed just before it hits the ground?

(let $y = 0$ at the base of the cliff)

$$U_i + K_i + W_{\text{nc}} = U_f + K_f$$

$$mgy + 1/2 mv_i^2 = (1/2)mv_f^2 \quad (\text{divide through by } m; \text{ multiply through by } 2)$$

$$2gy + v_i^2 = v_f^2$$

$$v_f = \sqrt{2 \left(9.8 \frac{\text{m}}{\text{s}^2} \right) (70 \text{ m}) + \left(26 \frac{\text{m}}{\text{s}} \right)^2}$$

$$v_f = 45.6 \text{ m/s}$$

12) A car driving at a speed of 20.0 m/s on level ground slams on its brakes. If it skids for 32.0 m before stopping, what is the coefficient of kinetic friction between its tires and the road?

$$U_i + K_i + W_{\text{nc}} = U_f + K_f$$

$$(1/2)mv^2 = -W_{\text{nc}}$$

$$(1/2)mv^2 = -f d (\cos \Theta)$$

$$(f = \mu mg; \quad \Theta = 180^\circ; \quad \Delta x = 32 \text{ m})$$

$$mv^2/2 = -(\mu mg)(-1)(\Delta x)$$

$$\mu = \frac{v^2}{2gd}$$

$$\mu = (20 \text{ m/s})^2 / (2)(9.8 \text{ m/s}^2)(32 \text{ m})$$

$$\mu = 0.638$$